Canguro Matemático Costarricense

Student Test
Expert Level

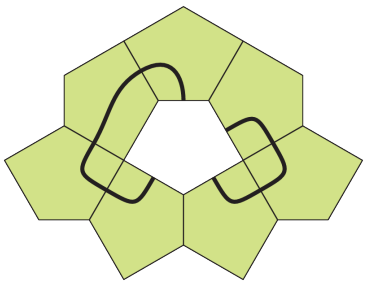
Name of the student:_____

Name of the institution:_____

Kangourou Sans Frontières
Costa Rica 2024

3 points

1. A pattern is made of equal pentagons.



Which of the tiles below, when placed in the central hole, will form a self-intersecting loop?

(**A**) 

(**B**) 

(**C**) 

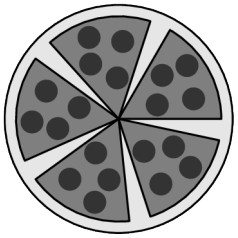
(**D**) 

(**E**) 

2. Which of these integers is two less than a multiple of ten, two more than a square, and two times a prime?

(**A**) 78 (**B**) 58 (**C**) 38 (**D**) 18 (**E**) 6

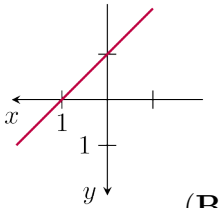
3. A young kangaroo cut a pizza into six equal slices. After eating one slice, he arranged the remaining slices with equal gaps between slices.

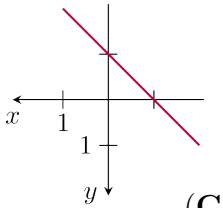


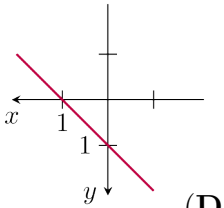
What size is the angle of each gap?

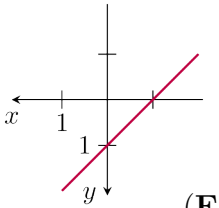
(**A**) 5° (**B**) 8° (**C**) 9°
(**D**) 10° (**E**) 12°

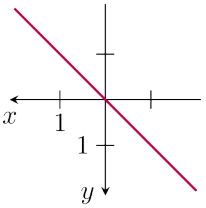
4. Juuso has an unusual habit of drawing the xy-plane with the positive coordinate axes pointing left and down. What would the graph of the equation $y = x + 1$ look like in a coordinate system drawn by Juuso?

(**A**) 

(**B**) 

(**C**) 

(**D**) 

(**E**) 

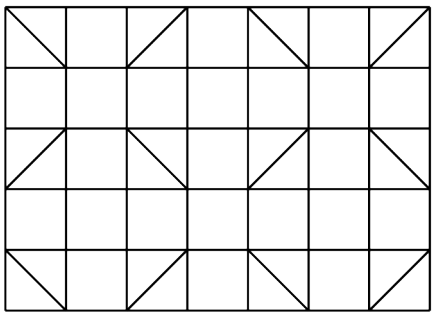
5. Kaito has manipulated a die. The probabilities of rolling a 2, 3, 4 or 5 are still $\frac{1}{6}$ each, but the probability of rolling a 6 is twice the probability of rolling a 1. What is the probability of rolling a 6?

(**A**) $\frac{1}{4}$ (**B**) $\frac{1}{6}$ (**C**) $\frac{7}{36}$ (**D**) $\frac{2}{9}$ (**E**) $\frac{5}{18}$

6. Which of the expressions below has the same value as $16^{15} + 16^{15} + 16^{15} + 16^{15}$?

(**A**) 16^{19} (**B**) 4^{31} (**C**) 4^{60} (**D**) 16^{60} (**E**) 4^{122}

7. Beaver wishes to color the squares and triangles of the following figure.



In such a way that no two neighbouring figures, even those sharing a single vertex, are the same color.

What is the least number of colors needed?

(**A**) 3 (**B**) 4 (**C**) 5 (**D**) 6 (**E**) 7

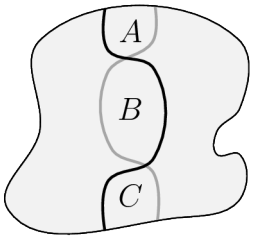
8. There are 6 glasses on a table with their open ends up. In any one move, we turn over exactly 4 of them. What is the least number of moves required to have all glasses upside down?

(**A**) 2 (**B**) 3 (**C**) 4 (**D**) 5 (**E**) 6

9. A student started with the number 1 and multiplied it by either 6 or 10. He then multiplied the result by either 6 or 10, and continued this procedure many times. Which of the following cannot be one of the numbers he obtained?

(**A**) $2^{100}3^{20}5^{80}$ (**B**) $2^{90}3^{20}5^{80}$ (**C**) $2^{90}3^{20}5^{70}$
(**D**) $2^{110}3^{80}5^{30}$ (**E**) $2^{50}5^{50}$

10. A black trail and a grey trail cross a park, as shown.



Each trail divides the park into two regions of equal area. Which of the following must be true about the areas A, B and C?

(**A**) $A = C$ (**B**) $B = A + C$ (**C**) $B = \frac{1}{2}(A + C)$

(**D**) $B = \frac{2}{3}(A + C)$ (**E**) $B = \frac{3}{5}(A + C)$

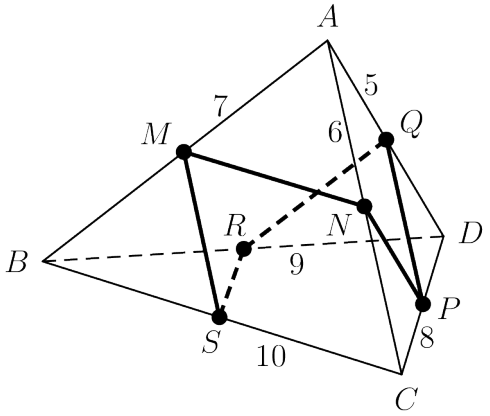
4 points

11. Exactly one of these statements about a certain positive integer n is true. Which statement is true?

(**A**) n is divisible by 3 (**B**) n is divisible by 6 (**C**) n is odd

(**D**) $n = 2$ (**E**) n is prime

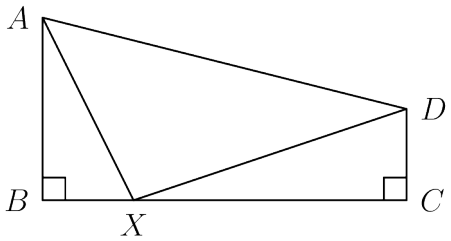
12. A triangular pyramid $ABCD$ has sides of length 5, 6, 7, 8, 9 and 10. The points M, N, P, Q, R and S are the midpoints of the edges of the pyramid, as shown.



What is the perimeter of the closed hexagonal line $MNPQRSM$?

(**A**) 19 (**B**) 20 (**C**) 21 (**D**) 22 (**E**) 23

13. A quadrilateral $ABCD$ has two right angles at B and C, where $AB = 4, BC = 8$ and $CD = 2$. Point X lies on BC.



What is the minimum value of $AX + DX$?

(**A**) $9\sqrt{2}$ (**B**) 12

(**C**) 13 (**D**) 10

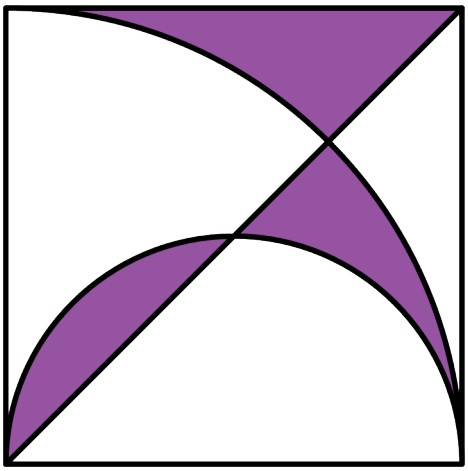
(**E**) None of the previous

14. John has a number of all black or all white unit cubes and wants to build a $3 \times 3 \times 3$ cube using 27 of them. He wants the surface to be exactly half black and half white. What is the smallest number of black cubes he can use?

(**A**) 14 (**B**) 13 (**C**) 12 (**D**) 11

(**E**) None of the previous

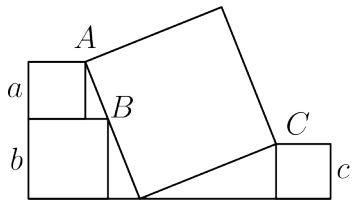
15. A diagonal, a semicircle and a quadrant are drawn in a square of side 6 cm.



What is the area, in cm^2, of the shaded part?

(**A**) 9 (**B**) 3π (**C**) $6\pi - 9$ (**D**) $10\pi/3$ (**E**) 12

16. The figure shows four squares.



The smaller ones have side lengths a, b and c. The vertices A and C of two of the smaller squares coincide with two diagonally opposite vertices of the large square. The vertex B of the third small square is on the side of the large one. Which of the following expressions represents the side length of the largest square?

(**A**) $\frac{1}{2}(a+b+c)$ (**B**) $\sqrt{a^2+b^2+c^2}$ (**C**) $\sqrt{(a+b)^2+c^2}$
(**D**) $\sqrt{(b-a)^2+c^2}$ (**E**) $\sqrt{a^2+ab+b^2+c^2}$

17. We have two positive numbers p and q, with $p < q$. Which of these expressions is the largest?

(**A**) $\frac{p+3q}{4}$ (**B**) $\frac{p+2q}{3}$ (**C**) $\frac{p+q}{2}$ (**D**) $\frac{2p+q}{3}$ (**E**) $\frac{3p+q}{4}$

18. How many three-digit numbers are there that contain at least one of the digits 1, 2 or 3?

(**A**) 27 (**B**) 147 (**C**) 441 (**D**) 557 (**E**) 606

19. I write down a 4-digit non-zero number $N = \overline{pqrs}$. When I place a decimal point between the q and the r, I find that the resulting number $\overline{pq.rs}$ is the average of the two-digit numbers $\overline{pq}$ and $\overline{rs}$.

What is the sum of the digits of N?

(**A**) 14 (**B**) 18 (**C**) 21 (**D**) 25 (**E**) 27

20. Two candles of equal length start burning at the same time. One of the candles will burn down in 4 hours, the other in 5 hours, each at their own constant rate. How many hours will they have to burn before one candle is 3 times the length of the other?

(**A**) $\frac{40}{11}$ (**B**) $\frac{45}{12}$ (**C**) $\frac{63}{20}$ (**D**) 3 (**E**) $\frac{47}{14}$

5 points

21. Andre has six cards with one number written on each side of each card. The pairs of numbers on the cards are $(5, 12), (3, 11), (0, 16), (7, 8), (4, 14)$ and $(9, 10)$.
The cards can be placed in any order in the blank spaces of the figure.

$$\square + \square + \square - \square - \square - \square = ?$$

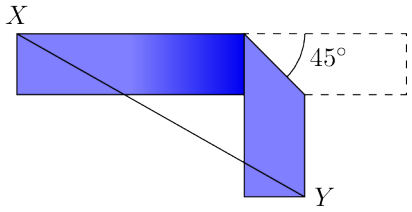
What is the smallest result he can get?

(**A**) -23 (**B**) -24 (**C**) -25 (**D**) -26 (**E**) -27

22. Kangaroo solves the equation $ax^2+bx+c = 0$, and Beaver solves the equation $bx^2+ax+c = 0$, where a, b, c are pairwise distinct non-zero integers. It turns out that the equations share a solution. Which of the following must be true?

(**A**) The common solution must be 0.
(**B**) The quadratic equation $ax^2 + bx + c = 0$ has exactly one real solution.
(**C**) $a > 0$
(**D**) $b < 0$
(**E**) $a + b + c = 0$

23. I have a strip of paper that is 12 cm long and 2 cm wide. I make a crease across it at 45° and then fold it, so that the two parts of the strip are aligned in a right angle, as shown.



What is the smallest possible length, in cm, of XY?

(**A**) $6\sqrt{2}$ (**B**) $7\sqrt{2}$ (**C**) 10 (**D**) 8 (**E**) $6 + \sqrt{2}$

24. Rasika has several unbiased 12-sided dice, each with faces labelled 1 to 12. When rolling all the dice at once, the probability of rolling a 12 exactly once is equal to the probability of rolling no 12s. How many dice does Rasika have?

(**A**) 8 (**B**) 9 (**C**) 10 (**D**) 11 (**E**) 12

25. A polynomial $p(x)$ satisfies the relation $p(x+1) = x^2 - x + 2p(6)$ for every real x. What is the sum of the coefficients of p?

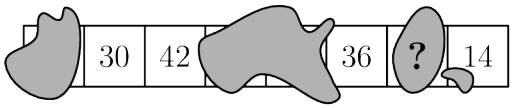
(**A**) -40 (**B**) -6 (**C**) 12 (**D**) 40

(**E**) None of the previous

26. The values of x, y and z satisfy $2^x = 3$, $2^y = 7$ and $6^z = 7$. Which of the following gives the relationship between x, y and z?

(**A**) $z = \dfrac{y}{1+x}$ (**B**) $z = \dfrac{x}{y} + 1$ (**C**) $z = \dfrac{y}{x} - 1$ (**D**) $z = \dfrac{x}{y-1}$ (**E**) $z = y - \dfrac{1}{x}$

27. A strip of paper consists of eight squares. Initially each square contains the number 0. In every move ,we chose 4 consecutive squares and add one to each of the numbers in those squares. The figure on the right shows the outcome after a number of moves but unfortunately some ink is covering some of the squares.



What number is written on the square with the question mark?

(**A**) 24 (**B**) 30 (**C**) 36 (**D**) 48

(**E**) None of the previous

28. A function $f\colon \mathbb{R} \longrightarrow \mathbb{R}$ satisfies $f(20 - x) = f(22 + x)$ for all real x. It is known that f has exactly two roots. What is the sum of these two roots?

(**A**) -1 (**B**) 20 (**C**) 21 (**D**) 22

(**E**) None of the previous

29. Twelve points are equally spaced on a circle. How many triangles containing a $45°$ angle can be formed by choosing three of these points?

(**A**) 48 (**B**) 60 (**C**) 72 (**D**) 84 (**E**) 96

30. A special four-digit number $\overline{abcd}$ satisfies the equation $\overline{abcd} = a^a + b^b + c^c + d^d$. What is the value of a?

(**A**) 2 (**B**) 3 (**C**) 4 (**D**) 5 (**E**) 6

Name:_____

Institution:_____

01.	A	B	C	D	E
02.	A	B	C	D	E
03.	A	B	C	D	E
04.	A	B	C	D	E
05.	A	B	C	D	E
06.	A	B	C	D	E
07.	A	B	C	D	E
08.	A	B	C	D	E
09.	A	B	C	D	E
10.	A	B	C	D	E
11.	A	B	C	D	E
12.	A	B	C	D	E
13.	A	B	C	D	E
14.	A	B	C	D	E
15.	A	B	C	D	E

16.	A	B	C	D	E
17.	A	B	C	D	E
18.	A	B	C	D	E
19.	A	B	C	D	E
20.	A	B	C	D	E
21.	A	B	C	D	E
22.	A	B	C	D	E
23.	A	B	C	D	E
24.	A	B	C	D	E
25.	A	B	C	D	E
26.	A	B	C	D	E
27.	A	B	C	D	E
28.	A	B	C	D	E
29.	A	B	C	D	E
30.	A	B	C	D	E